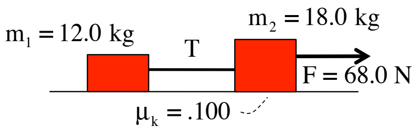
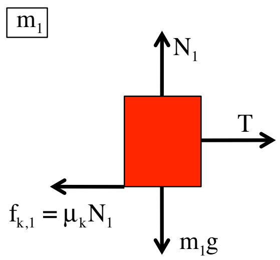
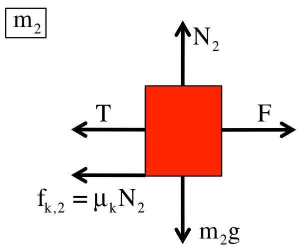
Problem 5.47

The systems is shown to the right.



a.) F.b.d. for each block:





Notice I've used the same variable ("T") on each f.b.d. Why? Because the forces are action/reaction pairs and, as a consequence, have the same magnitude. ALWAYS DO THIS with action/reaction pairs!

b.) Determine the acceleration of the system:

Quick and Dirty: Assuming the forces that make the system accelerate to the right are positive (which means that the acceleration of the system in this case will be positive and to the right), and noting by inspection that the normal force on in both cases is of the form "mg," dealing only with forces that actually accelerate the system yields:

$\sum F_{acc}$:

$$F - \mu_k N_1 - \mu_k N_2 = (m_1 + m_2)a$$

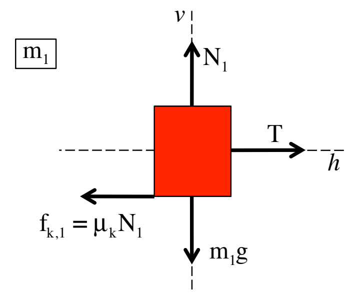
$$\Rightarrow \quad F - \mu_k(m_1 g) - \mu_k(m_2 g) = (m_1 + m_2)a$$

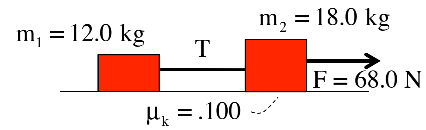
$$\Rightarrow \quad a = \frac{F - \mu_k(m_1 g) - \mu_k(m_2 g)}{m_1 + m_2}$$

Equation 1

Now let's see if the Formal approach gives us the same acceleration term.

Determine the acceleration of the system using the Formal approach:





$\sum F_v$:

$$N_1 - m_1 g = m_1 a \quad (a = 0)$$

$$\Rightarrow \quad N_1 = m_1 g$$

$\sum F_v$:

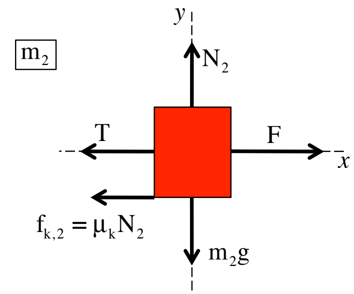
$$-\mu_k N_1 + T = m_1 a$$

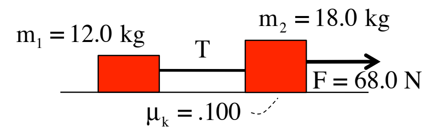
$$\Rightarrow \quad -\mu_k(m_1 g) + T = m_1 a$$

$$\Rightarrow \quad T = m_1 a + \mu_k(m_1 g)$$

Equation A

Determine the acceleration of the system using the Formal approach:





$\sum F_y$:

$$N_2 - m_2 g = m_2 a \quad (a = 0)$$

$$\Rightarrow \quad N_2 = m_2 g$$

$\sum F_x$:

$$-\mu_k N_2 - T + F = m_2 a$$

$$\Rightarrow \quad -\mu_k(m_2 g) - T + F = m_2 a$$

$$\Rightarrow \quad T = F - m_2 a - \mu_k(m_2 g)$$

Equation B

Using Equation A and B (equating the "T" terms), we get:

$$m_1a + \mu_k(m_1g) = F - m_2a - \mu_k(m_2g)$$

$$\Rightarrow \quad a = \frac{-F + \mu_k(m_1g) + \mu_k(m_2g)}{-m_1 - m_2} \qquad \text{Equation 2}$$

$$= \frac{-(68.0\text{ N}) + (.100)(12.0\text{ kg})(9.80\text{ m/s}^2) + (.100)(18.0\text{ kg})(9.80\text{ m/s}^2)}{-(12.0+18.0)\text{kg}}$$

$$= 1.29\text{ m/s}^2$$

Note 1: Notice that Equation 1 from the Quick and Dirty approach and Equation 2 from the Formal approach are the same (you have to multiply the top and bottom by -1 to see this, but that doesn't alter the observation). Yippee!

Note 2: Notice that there are negative signs in the denominator of Equation 2. This isn't a problem as long as the signs are the same. If, on the other hand, one sign was positive and one negative, then for two masses of equal size the denominator would go to zero. If that ever happens, it usually means you haven't unembedded a negative sign from one of the "ma" acceleration terms.

c.) From Equations A or B, the tension is found to be:

$$T_1 = m_1a + \mu_k(m_1g) \qquad \text{Equation A}$$

$$= (12.00\text{ kg})(1.29\text{ m/s}^2) + (.100)(12.0\text{ kg})(9.80\text{ m/s}^2)$$

$$= 27.2\text{ N}$$

or

$$T_1 = F - m_2a - \mu_k(m_2g) \qquad \text{Equation B}$$

$$= (68.0\text{ N}) - (18.00\text{ kg})(1.29\text{ m/s}^2) - (.100)(18.0\text{ kg})(9.80\text{ m/s}^2)$$

$$= 27.1\text{ N}$$

The two solutions are close enough for government work.